

Factorization of Soft and Collinear Divergences in Non-Equilibrium Quantum Field Theory

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Abstract

Proof of factorization of soft and collinear divergences in non-equilibrium QCD may be necessary to study hadronic signatures of quark-gluon plasma at RHIC and LHC. In this paper we prove factorization of soft and collinear divergences in non-equilibrium QED by using Schwinger-Keldysh closed-time path integral formalism in the background field method in pure gauge.

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I. INTRODUCTION

RHIC and LHC heavy-ion colliders are the best facilities to study quark-gluon plasma in the laboratory. Since two nuclei travel almost at speed of light, the QCD matter formed at RHIC and LHC may be in non-equilibrium. In order to make meaningful comparison of the theory with the experimental data on hadron production, it may be necessary to study nonequilibrium-nonperturbative QCD at RHIC and LHC. This, however, is a difficult problem.

Non-equilibrium quantum field theory can be studied by using Schwinger-Keldysh closed-time path (CTP) formalism [1, 2]. However, implementing CTP in non-equilibrium at RHIC and LHC is a very difficult problem, especially due to the presence of gluons in non-equilibrium and hadronization etc. Recently, one-loop resummed gluon propagator in non-equilibrium in covariant gauge is derived in [3, 4].

High p_T hadron production at high energy e^+e^- , ep and pp colliders is studied by using Collins-Soper fragmentation function [5, 6, 7]. For a high p_T parton fragmenting to hadron, Collins-Soper derived an expression for the fragmentation function based on field theory and factorization properties in QCD at high energy. This fragmentation function is universal in the sense that, once its value is determined from one experiment it explains the data at other experiments [8].

Recently we have derived parton-to-hadron fragmentation function in non-equilibrium QCD by using Schwinger-Keldysh closed-time path integral formalism [9]. This can be relevant at RHIC and LHC heavy-ion colliders to study hadron production from quark-gluon plasma. We have considered a high p_T parton in QCD medium at initial time τ_0 with arbitrary non-equilibrium (non-isotropic) distribution function $f(\vec{p})$, fragmenting to hadron. The special case $f(\vec{p}) = \frac{1}{e^{\frac{p_0}{T}} \pm 1}$ corresponds to the finite temperature QCD in equilibrium.

In order to study hadron production from quark-gluon plasma using the non-equilibrium fragmentation function we will need to prove factorization of fragmentation function in non-equilibrium QCD. Factorization refers to separation of short distance from long distance effects in field theory. In this paper we will prove the factorization of soft and collinear divergences in non-equilibrium QED. The proof of factorization of fragmentation function in non-equilibrium QCD at RHIC and LHC will be addressed elsewhere.

The paper is organized as follows. In section II we briefly describe factorization of soft

and collinear divergences in QED in vacuum. In section III we describe Schwinger-Keldysh closed-time path integral formalism in non-equilibrium quantum field theory relevant for our purpose. In section IV we prove factorization of soft and collinear divergences in non-equilibrium QED by using Schwinger-Keldysh closed-time path formalism in the background field method in pure gauge. Section V contains conclusions.

II. FACTORIZATION OF SOFT AND COLLINEAR DIVERGENCES IN QED IN VACUUM

The Wilson line is given by [6, 10]

$$\Phi[x^\mu] = \mathcal{P} \exp[-ie \int_{-\infty}^0 d\lambda h \cdot A(x^\mu + h^\mu \lambda)] \quad (1)$$

where h^μ is a x^μ independent four vector. Eq. (1) can be written as

$$\begin{aligned} \Phi[x^\mu] &= \mathcal{P} \exp[-ie \int_{-\infty}^0 d\lambda h \cdot A(x^\mu + h^\mu \lambda)] \\ &= \mathcal{P} \exp[-ie \int_{-\infty}^0 d\lambda h \cdot e^{\lambda h \cdot \partial} A(x^\mu)] = \mathcal{P} \exp[-ie \frac{1}{h \cdot \partial} h \cdot A(x^\mu)]. \end{aligned} \quad (2)$$

Since the vector h^μ is free we can choose it to correspond to various physical situations. For example if we choose $h^\mu = n^\mu$, where n^μ is a fixed lightlike vector, we find from eq. (2) the phase factor

$$\omega(x) = \frac{1}{n \cdot \partial} n \cdot A(x). \quad (3)$$

Using the Fourier transformation

$$A_\mu(x) = \int \frac{d^4 k}{(2\pi)^4} A_\mu(k) e^{ik \cdot x} \quad (4)$$

we find from eq. (3)

$$V = e \omega(k) = ie \frac{n^\mu}{n \cdot k} A_\mu(k). \quad (5)$$

Note that eq. (5) is precisely the eikonal vertex for a soft photon with momentum k interacting with a high energy electron jet moving along the direction n^μ [5, 6]. In the soft photon approximation in [6] the fixed lightlike vector is taken to be having only ”+” or ”-” component:

$$n^\mu = (n^+, n^-, n_T) = (1, 0, 0) \quad \text{or} \quad n^\mu = (n^+, n^-, n_T) = (0, 1, 0). \quad (6)$$

Now we will show that when the classical background field $A_\mu(x)$ is a pure gauge given by

$$A_\mu(x) = \partial_\mu \omega(x) \quad (7)$$

we can reproduce eqs. (3) and (5) which appear in the Wilson line eq. (2). Now multiplying a four vector h^μ from left in eq. (7) we find

$$h \cdot A(x) = h \cdot \partial \omega(x). \quad (8)$$

Dividing $h \cdot \partial$ from left in the above equation we find

$$\omega(x) = \frac{1}{h \cdot \partial} h \cdot A(x). \quad (9)$$

Since the four vector h^μ is free we can choose it such way that the pure gauge can represent Feynman rules involving soft photons or collinear photons. For example, when $h^\mu = n^\mu$ where n^μ is a fixed vector (see eq. (6)), we find from the above equation

$$\omega(x) = \frac{1}{n \cdot \partial} n \cdot A(x) \quad (10)$$

which reproduces eq. (3) which appears inside the Wilson line in eq. (2). Hence we have established the correspondence between the Wilson line and the classical background field $A_\mu(x)$ in pure gauge in the context of soft divergences.

Similarly, if we choose $h^\mu = n_B^\mu$, where n_B^μ is a non-light like vector

$$n_B^\mu = (n_B^+, n_B^-, 0), \quad (11)$$

we reproduce the Feynman rules for the collinear divergences [6]. This establishes the correspondence between the Wilson line and the classical background field $A_\mu(x)$ in pure gauge in the context of collinear divergences.

The generating functional in QED in the presence of background field $A_\mu(x)$ is given by [10]

$$Z[A, J, \eta, \bar{\eta}] = \int [dQ][d\bar{\psi}][d\psi] e^{i \int d^4x [-\frac{1}{4}F_{\mu\nu}^2[Q] - \frac{1}{2\alpha}(\partial_\mu Q^\mu)^2 + \bar{\psi} D[A+Q]\psi + J \cdot Q + \bar{\eta}\psi + \eta\bar{\psi}]} \quad (12)$$

where

$$D[A+Q] = (i\partial - eQ - eA). \quad (13)$$

For the purpose of studying soft and collinear divergences it is understood that the integration in $Z[A, J, \eta, \bar{\eta}]$ is only over those functions of the quantum photon field $Q(x)$, electron field $\psi(x)$ and positron field $\bar{\psi}(x)$ such that their Fourier transforms $Q(k)$, $\psi(k)$ and $\bar{\psi}(k)$ vanish in the soft region. Under the following transformation of the fermion fields and the sources

$$\psi' = U \psi, \quad \bar{\psi}' = \bar{\psi} U^{-1}, \quad \eta' = U \eta, \quad \bar{\eta}' = \bar{\eta} U^{-1}, \quad U = e^{-ie\omega(x)} \quad (14)$$

we find from eq. (12)

$$Z[A, J, \eta', \bar{\eta}'] = Z[J, \eta, \bar{\eta}]. \quad (15)$$

The correlation function in QED in the presence of background field $A_\mu(x)$ is given by

$$\frac{\delta}{\delta\bar{\eta}(x_2)} \frac{\delta}{\delta\eta(x_1)} Z[A, J, \eta, \bar{\eta}] |_{J=\eta=\bar{\eta}=0} = \langle \psi(x_2) \bar{\psi}(x_1) \rangle_A. \quad (16)$$

Similarly the correlation function in QED (without the background field) is given by

$$\frac{\delta}{\delta\bar{\eta}(x_2)} \frac{\delta}{\delta\eta(x_1)} Z[J, \eta, \bar{\eta}] |_{J=\eta=\bar{\eta}=0} = \langle \psi(x_2) \bar{\psi}(x_1) \rangle_{A=0}. \quad (17)$$

Hence we find from eqs. (15), (16) and (17)

$$\langle \psi(x_2) \bar{\psi}(x_1) \rangle_A = e^{-ie\omega(x_2)} \langle \psi(x_2) \bar{\psi}(x_1) \rangle_{A=0} e^{ie\omega(x_1)} \quad (18)$$

where all the $A_\mu(x)$ dependence has been factored into $e^{-ie\omega(x_2)}$ and $e^{ie\omega(x_1)}$. Using eqs. (9) and (2) we find

$$\begin{aligned} \langle \psi(x_2) \bar{\psi}(x_1) \rangle_A &= \exp[-ie \int_{-\infty}^0 d\lambda h \cdot A(x_2^\mu + h^\mu \lambda)] \\ &\times \langle \psi(x_2) \bar{\psi}(x_1) \rangle_{A=0} \times \exp[-ie \int_0^{-\infty} d\lambda h \cdot A(x_1^\nu + h^\nu \lambda)]. \end{aligned} \quad (19)$$

This proves factorization of soft and collinear divergences in QED [10].

III. SCHWINGER-KELDysh CLOSED-TIME PATH INTEGRAL FORMALISM IN NON-EQUILIBRIUM QUANTUM FIELD THEORY

Unlike pp collisions, the ground state at RHIC and LHC heavy-ion collisions (due to the presence of a QCD medium at initial time $t = t_{in}$ (say $t_{in}=0$) is not a vacuum state $|0\rangle$ any

more. We denote $|in\rangle$ as the initial state of the non-equilibrium QCD medium at t_{in} . For example, if the system at RHIC and LHC at initial time is space translational invariance the non-equilibrium distribution function $f(\vec{k})$ of a parton (quark or gluon), corresponding to such initial state can be written as

$$\langle a^\dagger(\vec{k})a(\vec{k}') \rangle = \langle in|a^\dagger(\vec{k})a(\vec{k}')|in \rangle = f(\vec{k})(2\pi)^{d-1}\delta^{(d-1)}(\vec{k} - \vec{k}'). \quad (20)$$

Finite temperature field theory formulation is a special case of this when $f(\vec{k}) = \frac{1}{e^{\frac{k_0}{T}} \pm 1}$.

Consider the time evolution of the density matrix

$$i\frac{\partial \rho}{\partial t} = [H_I, \rho], \quad \rho(-\infty) = \rho_0 \quad (21)$$

where H_I is the interaction hamiltonian. The formal solution is

$$\rho(t) = S(t, -\infty)\rho_0 S(-\infty, t), \quad S(t, -\infty) = T \exp[-i \int_{-\infty}^t dt' H_I(t')]. \quad (22)$$

The density matrix ρ is in interaction picture. The average value of an operator L in the interaction picture is given by

$$\langle L(t) \rangle = Tr[\rho(t)L(t)], \quad i\frac{\partial L(t)}{\partial t} = [H_0, L(t)] \quad (23)$$

where H_0 is the free hamiltonian. Since in many situations we deal with correlation function of several fields at different times, it is useful to transfer all the time dependence to field operators and consider the density operator as independent of time, *i.e.* to go to Heisenberg representation. For the time independence of the density matrix we can take the value of the matrix determined by expression eq. (22) at a certain fixed instant of time, for example, $t = t_{in} = 0$, having thus included in it all the changes which the distribution ρ_0 had undergone when the external field and the interaction in the system were switched on.

Consider the medium average of T -products of several operators. Using the Heisenberg density matrix one finds

$$\begin{aligned} \langle T[L(t)M(t')...] \rangle &= Tr[\rho T[L(t)M(t')....]] = Tr[S(0, -\infty)\rho_0 S(-\infty, 0)T[L(t)M(t')....]] \\ &= Tr[\rho_0 S(-\infty, 0)T[L(t)M(t')....]S(0, -\infty)]. \end{aligned} \quad (24)$$

Going over to the operators in the interaction picture

$$\begin{aligned} \langle T[L(t)M(t')...] \rangle &= Tr[\rho_0 S(-\infty, 0)T[S(0, t)L_I(t)S(t, t')M_I(t')....]S(0, -\infty)] \\ &= Tr[\rho_0 T_c[S_c L_I(t)M_I(t')....]] \end{aligned} \quad (25)$$

where T_c is the complete contour from

$$-\infty \rightarrow t \rightarrow t' \dots \rightarrow -\infty \quad (26)$$

and S_c is the complete S -matrix defined along T_c .

To deal with Feynman diagram and Wick theorem it is useful to split the time interval to "+" and "-" contour; where "+" time branch is from $-\infty$ to $+\infty$ where (time) T -order product apply and "-" time branch is from $+\infty$ to $-\infty$ where (anti-time) \bar{T} -order product apply.

A. Generating Functional in Non-Equilibrium Scalar Field Theory

Consider scalar field theory first. Since there are two time branches there are two fields and two sources and hence four Green's functions. Let us denote the field $\phi_+(x)$ and the source $J_+(x)$ in the "+" time branch and $\phi_-(x)$ and $J_-(x)$ in the "-" time branch. The generating functional is given by

$$Z[J_+, J_-, \rho] = \int [d\phi_+] [d\phi_-] \exp[iS[\phi_+] - S[\phi_-] + \int d^4x J_+ \phi_+ - \int d^4x J_- \phi_-] \langle \phi_+, 0 | \rho | 0, \phi_- \rangle \quad (27)$$

where $S[\phi]$ is the full action in scalar field theory and $|\phi_\pm, 0\rangle$ is the quantum state corresponding to the field configuration $\phi_\pm(\vec{x}, t=0)$.

In the CTP formalism in non-equilibrium there are four Green's functions

$$\begin{aligned} G_{++}(x, x') &= \frac{\delta Z[J_+, J_-, \rho]}{i^2 \delta J_+(x) J_+(x')} = \langle in | T \phi(x) \phi(x') | in \rangle = \langle T \phi(x) \phi(x') \rangle \\ G_{--}(x, x') &= \frac{\delta Z[J_+, J_-, \rho]}{(-i)^2 \delta J_-(x) J_-(x')} = \langle in | \bar{T} \phi(x) \phi(x') | in \rangle = \langle \bar{T} \phi(x) \phi(x') \rangle \\ G_{+-}(x, x') &= \frac{\delta Z[J_+, J_-, \rho]}{-i^2 \delta J_+(x) J_-(x')} = \langle in | \phi(x') \phi(x) | in \rangle = \langle \phi(x') \phi(x) \rangle \\ G_{-+}(x, x') &= \frac{\delta Z[J_+, J_-, \rho]}{-i^2 \delta J_-(x) J_+(x')} = \langle in | \phi(x) \phi(x') | in \rangle = \langle \phi(x) \phi(x') \rangle \end{aligned} \quad (28)$$

where T is the time order product and \bar{T} is the anti-time order product given by

$$\begin{aligned} T \phi(x) \phi(x') &= \theta(t - t') \phi(x) \phi(x') + \theta(t' - t) \phi(x') \phi(x) \\ \bar{T} \phi(x) \phi(x') &= \theta(t' - t) \phi(x) \phi(x') + \theta(t - t') \phi(x') \phi(x). \end{aligned} \quad (29)$$

B. Generating Functional in Non-Equilibrium QED

Extending the same analysis to QED we find the generating functional in non-equilibrium QED

$$\begin{aligned}
Z[J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] &= \int [dQ_+][dQ_-][d\bar{\psi}_+][d\bar{\psi}_-][d\psi_+][d\psi_-] \times \\
&\exp[i \int d^4x [-\frac{1}{4}(F_{\mu\nu}^2[Q_+] - F_{\mu\nu}^2[Q_-]) - \frac{1}{2\alpha}((\partial_\mu Q_+^\mu)^2 - (\partial_\mu Q_-^\mu)^2) + \bar{\psi}_+ D[Q_+]\psi_+ \\
&- \bar{\psi}_- D[Q_-]\psi_- + J_+ \cdot Q_+ - J_- \cdot Q_- + \bar{\eta}_+ \psi_+ - \bar{\eta}_- \psi_- + \eta_+ \bar{\psi}_+ - \eta_- \bar{\psi}_-]] \\
&\times \langle Q_+, \psi_+, \bar{\psi}_+, 0 | \rho | 0, \bar{\psi}_-, \psi_-, Q_- \rangle.
\end{aligned} \tag{30}$$

where Q_μ is the photon field and J_μ is the corresponding source, ψ is the electron field and $\bar{\eta}$ is the corresponding source. ρ is the initial density of state. The state $|Q_\pm, \psi_\pm, \bar{\psi}_\pm, 0 \rangle$ is the quantum state corresponding to the field configurations $Q_\mu(\vec{x}, t = t_{in} = 0)$, $\psi(\vec{x}, t = t_{in} = 0)$ and $\bar{\psi}(\vec{x}, t = t_{in} = 0)$ respectively.

IV. FACTORIZATION OF SOFT AND COLLINEAR DIVERGENCES IN NON-EQUILIBRIUM QED

The generating functional in the background field method of QED is given by eq. (12). Extending this to non-equilibrium QED we find

$$\begin{aligned}
Z[\rho, A, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] &= \int [dQ_+][dQ_-][d\bar{\psi}_+][d\bar{\psi}_-][d\psi_+][d\psi_-] \times \\
&\exp[i \int d^4x [-\frac{1}{4}(F_{\mu\nu}^2[Q_+] - F_{\mu\nu}^2[Q_-]) - \frac{1}{2\alpha}((\partial_\mu Q_+^\mu)^2 - (\partial_\mu Q_-^\mu)^2) + \bar{\psi}_+ D[Q_+ + A_+]\psi_+ \\
&- \bar{\psi}_- D[Q_- + A_-]\psi_- + J_+ \cdot Q_+ - J_- \cdot Q_- + \bar{\eta}_+ \psi_+ - \bar{\eta}_- \psi_- + \eta_+ \bar{\psi}_+ - \eta_- \bar{\psi}_-]] \\
&\times \langle Q_+, \psi_+, \bar{\psi}_+, 0 | \rho | 0, \bar{\psi}_-, \psi_-, Q_- \rangle
\end{aligned} \tag{31}$$

where ρ is the initial density of states. The fermion fields and the sources transform as

$$\begin{aligned}
\psi'_+ &= U_+ \psi_+, & \bar{\psi}'_+ &= \bar{\psi}_+ U_+^{-1}, & \eta'_+ &= U_+ \eta_+, & \bar{\eta}'_+ &= \bar{\eta}_+ U_+^{-1} \\
\psi'_- &= U_- \psi_-, & \bar{\psi}'_- &= \bar{\psi}_- U_-^{-1}, & \eta'_- &= U_- \eta_-, & \bar{\eta}'_- &= \bar{\eta}_- U_-^{-1}
\end{aligned} \tag{32}$$

where

$$U_+ = e^{-ie\omega_+(x)}, \quad U_- = e^{-ie\omega_-(x)}. \tag{33}$$

Changing $\psi_{\pm} \rightarrow \psi'_{\pm}$, $\bar{\psi}_{\pm} \rightarrow \bar{\psi}'_{\pm}$ but keeping Q_{\pm} fixed we find from eq. (31)

$$\begin{aligned}
Z[\rho, A, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] &= \int [dQ_+][dQ_-][d\bar{\psi}_+][d\bar{\psi}_-][d\psi_+][d\psi_-] \times \\
&\exp[i \int d^4x [-\frac{1}{4}(F_{\mu\nu}^2[Q_+] - F_{\mu\nu}^2[Q_-]) - \frac{1}{2\alpha}((\partial_\mu Q_+^\mu)^2 - (\partial_\mu Q_-^\mu)^2) + \bar{\psi}_+ D[Q_+] \psi_+ - \bar{\psi}_- D[Q_-] \psi_- \\
&+ J_+ \cdot Q_+ - J_- \cdot Q_- + \bar{\eta}_+ \psi'_+ - \bar{\eta}_- \psi'_- + \eta_+ \bar{\psi}'_+ - \eta_- \bar{\psi}'_-]] \\
&\times \langle Q_+, \psi'_+, \bar{\psi}'_+, 0 | \rho | 0, \bar{\psi}_-, \psi'_-, Q_- \rangle.
\end{aligned} \tag{34}$$

Note that the state $|0, \bar{\psi}_-, \psi'_-, Q_- \rangle$ is at initial time $t = t_{in} = 0$. We find

$$\begin{aligned}
Z[\rho, A, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] &= \int [dQ_+][dQ_-][d\bar{\psi}_+][d\bar{\psi}_-][d\psi_+][d\psi_-] \times \\
&\exp[i \int d^4x [-\frac{1}{4}(F_{\mu\nu}^2[Q_+] - F_{\mu\nu}^2[Q_-]) - \frac{1}{2\alpha}((\partial_\mu Q_+^\mu)^2 - (\partial_\mu Q_-^\mu)^2) + \bar{\psi}_+ D[Q_+] \psi_+ - \bar{\psi}_- D[Q_-] \psi_- \\
&+ J_+ \cdot Q_+ - J_- \cdot Q_- + \bar{\eta}_+ \psi'_+ - \bar{\eta}_- \psi'_- + \eta_+ \bar{\psi}'_+ - \eta_- \bar{\psi}'_-]] \\
&\times \langle Q_+, \psi_+, \bar{\psi}_+, 0 | \rho | 0, \bar{\psi}_-, \psi_-, Q_- \rangle
\end{aligned} \tag{35}$$

which gives by using eq. (32)

$$\begin{aligned}
Z[\rho, A, J_+, J_-, \eta'_+, \eta'_-, \bar{\eta}'_+, \bar{\eta}'_-] &= \int [dQ_+][dQ_-][d\bar{\psi}_+][d\bar{\psi}_-][d\psi_+][d\psi_-] \times \\
&\exp[i \int d^4x [-\frac{1}{4}(F_{\mu\nu}^2[Q_+] - F_{\mu\nu}^2[Q_-]) - \frac{1}{2\alpha}((\partial_\mu Q_+^\mu)^2 - (\partial_\mu Q_-^\mu)^2) + \bar{\psi}_+ D[Q_+] \psi_+ - \bar{\psi}_- D[Q_-] \psi_- \\
&+ J_+ \cdot Q_+ - J_- \cdot Q_- + \bar{\eta}_+ \psi_+ - \bar{\eta}_- \psi_- + \eta_+ \bar{\psi}_+ - \eta_- \bar{\psi}_-]] \\
&\times \langle Q_+, \psi_+, \bar{\psi}_+, 0 | \rho | 0, \bar{\psi}_-, \psi_-, Q_- \rangle
\end{aligned} \tag{36}$$

Hence we find from eq. (36)

$$Z[\rho, A, J_+, J_-, \eta'_+, \eta'_-, \bar{\eta}'_+, \bar{\eta}'_-] = Z[\rho, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] \tag{37}$$

in non-equilibrium QED in the presence of background field in pure gauge.

The correlation functions in non-equilibrium QED in the presence of background field $A_\mu(a)$ is given by

$$\begin{aligned}
&\frac{\delta}{\delta \bar{\eta}_r(x_2)} \frac{\delta}{\delta \eta_s(x_1)} Z[\rho, A, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] \big|_{J_+=J_-=\eta_+=\eta_-=\bar{\eta}_+=\bar{\eta}_-=0} \\
&= \langle \psi_r(x_2) \bar{\psi}_s(x_1) \rangle_A = \langle in | \psi_r(x_2) \bar{\psi}_s(x_1) | in \rangle_A
\end{aligned} \tag{38}$$

where $r, s = +, -$ are the closed-time path indices. The correlation functions in non-equilibrium QED is given by

$$\begin{aligned}
&\frac{\delta}{\delta \bar{\eta}_r(x_2)} \frac{\delta}{\delta \eta_s(x_1)} Z[\rho, J_+, J_-, \eta_+, \eta_-, \bar{\eta}_+, \bar{\eta}_-] \big|_{J_+=J_-=\eta_+=\eta_-=\bar{\eta}_+=\bar{\eta}_-=0} \\
&= \langle \psi_r(x_2) \bar{\psi}_s(x_1) \rangle_{A=0} = \langle in | \psi_r(x_2) \bar{\psi}_s(x_1) | in \rangle_{A=0}.
\end{aligned} \tag{39}$$

Hence we find from eqs. (37), (38) and (39)

$$< in | \psi_r(x_2) \bar{\psi}_s(x_1) | in >_A = e^{-ie\omega_r(x_2)} < in | \psi_r(x_2) \bar{\psi}_s(x_1) | in >_{A=0} e^{ie\omega_s(x_1)}. \quad (40)$$

where all the $A_\mu(x)$ dependence has been factored into $e^{-ie\omega_r(x_2)}$ and $e^{ie\omega_s(x_1)}$. Using eqs. (9) and (2) we find

$$\begin{aligned} < in | \psi_r(x_2) \bar{\psi}_s(x_1) | in >_A = \exp[-ie \int_{-\infty}^0 d\lambda h \cdot A_r(x_2^\mu + h^\mu \lambda)] \times \\ < in | \psi_r(x_2) \bar{\psi}_s(x_1) | in >_{A=0} \times \exp[-ie \int_0^{-\infty} d\lambda h \cdot A_s(x_1^\nu + h^\nu \lambda)] \end{aligned} \quad (41)$$

which proves factorization of soft and collinear divergences in non-equilibrium QED. This is similar to eq. (19) except that the closed-time path indices $r, s = +, -$ have appeared in eq. (41). Note that the repeated indices r and s in eq. (41) are not summed.

V. CONCLUSIONS

In order to study hadron production from quark-gluon plasma using the non-equilibrium fragmentation function [9] we will need to prove factorization in non-equilibrium QCD. Factorization refers to separation of short distance from long distance effects in field theory. However, before proving factorization in non-equilibrium QCD we need to show a similar factorization in non-equilibrium QED. In this paper we have proved factorization of soft and collinear divergences in non-equilibrium QED by using Schwinger-Keldysh closed-time path integral formalism in the background field method of QED in pure gauge.

The proof of factorization of fragmentation function in non-equilibrium QCD at RHIC and LHC will be addressed elsewhere. This may be relevant to study hadron production [9] from quark-gluon plasma [11] at RHIC and LHC.

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